

# A brief introduction to Luttinger liquids

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**Abstract.** I give a brief introduction to Luttinger liquids. Luttinger liquids are paramagnetic one-dimensional metals without Landau quasi-particle excitations. The elementary excitations are collective charge and spin modes, leading to charge-spin separation. Correlation functions exhibit power-law behavior. All physical properties can be calculated, e.g. by bosonization, and depend on three parameters only: the renormalized coupling constant  $K_\rho$ , and the charge and spin velocities. I also discuss the stability of Luttinger liquids with respect to temperature, interchain coupling, lattice effects and phonons, and list important open problems.

## WHAT IS A LUTTINGER LIQUID ANYWAY?

Ordinary, three-dimensional metals are described by Fermi liquid theory. Fermi liquid theory is about the importance of electron-electron interactions in metals. It states that there is a 1:1-correspondence between the low-energy excitations of a free Fermi gas, and those of an interacting electron liquid which are termed “quasi-particles” [1]. Roughly speaking, the combination of the Pauli principle with low excitation energy (e.g.  $T \ll E_F$ ) and the large phase space available in 3D, produces a *very dilute* gas of excitations where interactions are sufficiently harmless so as to preserve the correspondence to the free-electron excitations. Three key elements are: (i) The elementary excitations of the Fermi liquid are quasi-particles. They lead to a pole structure (with residue  $Z$  – the overlap of a Fermi surface electron with free electrons) in the electronic Green’s function which can be – and has been – observed by photoemission spectroscopy [2]. (ii) Transport is described by the Boltzmann equation which, in favorable cases, can be quantitatively linked to the photoemission response [2]. (iii) The low-energy physics is parameterized by a set of Landau parameters  $F_{s,a}^\ell$  which contain the residual interaction effects in the angular momentum charge and spin channels. *The correlations in the electron system are weak, although the interactions may be very strong.*

Fermi liquid theory breaks down for one-dimensional (1D) metals. Technically, this happens because some vertices Fermi liquid theory assumes finite (those involving a  $2k_F$  momentum transfer) actually diverge because of the Peierls effect. An equivalent intuitive argument is that in 1D, perturbation theory never can work even for arbitrarily small but finite interactions: when degenerate perturbation

theory is applied to the coupling of the all-important electron states at the Fermi points  $\pm k_F$ , it will split them and therefore remove the entire Fermi surface! A free-electron-like metal will therefore not be stable in 1D. The underlying physical picture is that the coupling of quasi-particles to collective excitations is small in 3D but large in 1D, no matter how small the interaction. *Correlations are strong even for weak interactions!*

1D metals are described as Luttinger liquids [3,4]. *A Luttinger liquid is a paramagnetic one-dimensional metal without Landau quasi-particle excitations.* “Paramagnetic” and “metal” require that the spin and charge excitations are gapless, more precisely with dispersions  $\omega_\nu \approx v_\nu |q|$  ( $\nu = \rho, \sigma$  for charge and spin). Only when this requirement is fulfilled, a Luttinger liquid can form. The charge and spin modes (holons and spinons) possess different excitation energies  $v_\rho \neq v_\sigma$  and are bosons. This leads to the separation of charge and spin of an electron (or hole) added to the Fermi sea, in space-time, or  $q - \omega$ -space. Charge-spin separation prohibits quasi-particles: The pole structure of the Green’s function is changed to branch cuts, and therefore the quasi-particle residue  $Z$  is zero. Charge-spin separation in space-time can be nicely observed in computer simulations [5].

The bosonic nature of charge and spin excitations, together with the reduced dimensionality leads to a peculiar kind of short-range order at  $T = 0$ . The system is at a (quantum) critical point, with power-law correlations, and the scaling relations between the exponents of its correlation functions are parameterized by renormalized coupling constants  $K_\nu$ . The individual exponents are non-universal, i.e. depend on the interactions. For Luttinger liquids,  $K_\nu$  is the equivalent of the Landau parameters. As an example, the momentum distribution function  $n(k) \sim (k_F - k)^\alpha$  for  $k \approx k_F$  with  $\alpha = (K_\rho + K_\rho^{-1} - 2)/4$ . This directly illustrates the absence of quasi-particles: In a Fermi liquid,  $n(k)$  has a jump at  $k_F$  with amplitude  $Z$ .

## BOSONIZATION, OR HOW TO SOLVE THE 1D MANY-BODY PROBLEM BY HARMONIC OSCILLATORS

The appearance of charge and spin modes as stable, elementary excitations in 1D fermion systems can be rationalized from the spectrum of allowed particle-hole excitations. In 1D, low-energy particle-hole pairs with momenta between 0 and  $2k_F$  are not allowed, and for  $q \rightarrow 0$ , the range of allowed excitations shrinks to a one-parameter spectrum  $\omega_\nu \approx v_\nu |q|$ , indicating stable particles (cf. Fig. 1). True bosons are then obtained as linear combinations of these particle-hole excitations with a definite momentum  $q$ . Most importantly, we now can rewrite any interacting fermion Hamiltonian, provided its charge and spin excitations are gapless, as a harmonic oscillator and find an operator identity allowing to express fermion operators as functions of these bosons. This is the complete bosonization program.

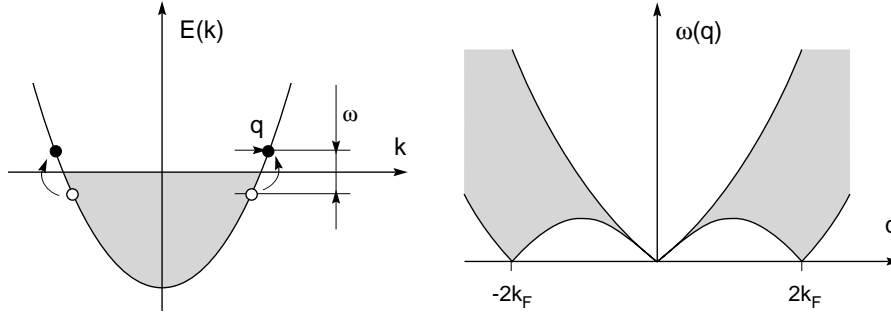
For free fermions, the Hamiltonian describing the *excitations* out of the ground state (the Fermi sea), can be expressed as a bilinear in the bosons,

$$H = \sum_{\nu=\rho,\sigma} \sum_q v_\nu |q| \left( b_{\nu,q}^\dagger b_{\nu,q} + 1/2 \right) , \quad (1)$$

with  $v_\nu = v_F$ , the Fermi velocity. Both the spectrum and the multiplicities of the states, i.e. the Hilbert space, of the fermion and boson forms are identical [3].

What happens in the presence of interactions? One possibility is that the interactions open a gap in the spin and/or charge excitation spectrum. The system then no longer is paramagnetic and/or metallic. With a charge gap, we have a 1D Mott insulator, with a spin gap a conducting system with strong charge density wave or superconducting correlations, and gaps in both channels imply a band insulator. Luttinger liquid theory cannot be applied anymore. In the other case, charge and spin excitations remain gapless: a Luttinger liquid is formed. Then, electron-electrons interactions will make  $v_\sigma \neq v_\rho \neq v_F$ , leading to charge-spin separation. Interactions will also renormalize the electronic compressibility and magnetic susceptibility, and the charge and spin stiffnesses, and by comparing the velocities measuring this renormalization to  $v_\nu$ , the correlation exponents  $K_\nu$  can be defined. The  $K_\nu$  therefore only depend on the low-energy properties of the Hamiltonian. Two parameters per degree of freedom,  $K_\nu$  and  $v_\nu$ , completely describe the physics of a Luttinger liquid.

From model studies, e.g. on the 1D Hubbard model [6] and related models [4], the following picture emerges: (i)  $K_\nu = 1$  describes free electrons, and  $K_\sigma = 1$  is required by spin-rotation invariance. (ii)  $K_\rho > 1$  for effectively attractive interactions, and  $K_\rho < 1$  for repulsive interactions. (iii) For the 1D Hubbard model,  $K_\rho$  decreases from 1 to 1/2 as the electron repulsion  $U$  varies between  $0 \dots \infty$ . (iv)  $K_\rho < 1$  decreases with increasing interaction range. For any finite range, there is a characteristic minimal  $K_\rho$ , which approaches zero, as the interaction range extends to infinity. (v)  $v_\sigma \leq v_F$  for repulsive interactions.  $v_\sigma$  measures the magnetic exchange  $J$ . (vi)  $v_\rho > v_F$  for repulsive interactions, and the more so the longer the interaction range. In the limit of unscreened Coulomb interaction,  $v_\rho \rightarrow \infty$ , and the charge fluctuations then become the 1D plasmons [7] with  $\omega_\rho(q) \propto |q| \ln |q|$ . (vii) Electron-phonon interaction decreases the  $v_\nu$ , and most often also  $K_\rho$ . Interaction



**FIGURE 1.** Particle-hole excitations in 1D (left). The spectrum of allowed excitation has no low-energy states with  $0 \leq |q| \leq 2k_F$ .

with high-frequency dispersionless molecular vibrations can enhance  $K_\rho$  and lead to superconductivity [8].

To complete our bosonization program, a local fermion operator must be expressed in terms of bosons. Exact operator identities are available for the Luttinger model [3,4] which can be summarized schematically as

$$\Psi_s(x) \sim \exp \left\{ i \sum_\nu \sum_p e^{ipx} (\dots) (b_{\nu,p} + b_{\nu,-p}^\dagger) \right\} . \quad (2)$$

This fermion-boson transformation turns bosonization into a useful device: all correlation functions can be calculated as simple harmonic oscillator averages, repeatedly using the two important identities  $e^A e^B = e^{A+B} e^{[A,B]/2}$  for  $[A, B]$  a complex number, and  $\langle e^A \rangle = \exp(\langle A^2 \rangle / 2)$  valid for harmonic oscillator expectation values. As a consequence, *Luttinger liquid predictions for all physical properties can be produced*. Examples are given in the next section. [The behavior of the momentum distribution function  $n(k)$  discussed above, has been obtained from the single-particle Green's function  $\langle T \Psi(xt) \Psi^\dagger(00) \rangle$  in precisely this way].

Bosonization is an easy and transparent way to calculate the properties of Luttinger liquids. However, it is not the only method. More general, and more powerful is the direct application of conformal field theory to a microscopic model of interacting fermions. For Luttinger liquids, both methods become identical, and one might view bosonization as solid state physicist's way of doing conformal field theory. Also Green's functions methods have been used successfully.

## PREDICTIONS FOR EXPERIMENTS

This section summarizes some important properties of Luttinger liquids in the form of experimental predictions. The underlying theoretical correlation functions can be found elsewhere [4]. We discuss a single-band Luttinger liquid. For multi-band systems, such as the metallic carbon nanotubes, the exponents differ from those give here, but can be calculated in the same way [9].

The thermodynamics is not qualitatively different from a Fermi liquid, with a linear-in- $T$  specific heat (expected both for 1D fermions and bosons!), and  $T$ -independent Pauli susceptibility and electronic compressibility

$$C(T) = \frac{1}{2} \left( \frac{v_F}{v_\rho} + \frac{v_F}{v_\sigma} \right) \gamma_0 T , \quad \chi = \frac{2K_\sigma}{\pi v_\sigma} , \quad \kappa = \frac{2K_\rho}{\pi v_\rho} . \quad (3)$$

More interesting are the charge and spin correlations at wavenumber multiples of  $k_F$  which display the  $K_\rho$ -dependent power laws discussed above. In the electronic structure factor  $S(k)$  and NMR spin-lattice relaxation rate  $T_1^{-1}$ , they translate into

$$S(k) \sim |k - 2k_F|^{K_\rho} + |k - 4k_F|^{4K_\rho - 1} , \quad T_1^{-1} \sim T + T^{K_\rho} . \quad (4)$$

The structure factor can be interpreted as showing fluctuations both of Peierls-type ( $2k_F$ ) and of Wigner-crystal-type ( $4k_F$ ) charge density waves, and the two terms in  $T_1^{-1}$  come from the  $q \approx 0$  and  $2k_F$  spin fluctuations. Evidence for such behavior has been found, e.g. in TTF-TCNQ [10] for  $S(k)$ , and  $(TMTSF)_2ClO_4$  [11] for  $T_1^{-1}$ . Transport properties depend on the scattering mechanisms assumed. If we consider electron-electron scattering in a band with filling factor  $1/n$ , we obtain from the current-current correlations [12]

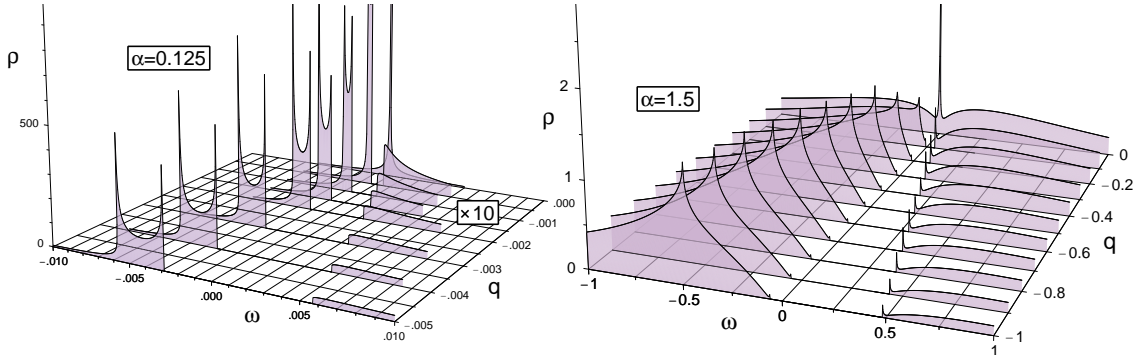
$$\rho(T) \sim T^{n^2 K_\rho - 3}, \quad \sigma(\omega) \sim \omega^{n^2 K_\rho - 5}. \quad (5)$$

The second law has apparently been observed in salts based on TMTSF [13]. These predictions ideally give information on the power-law behavior of correlations, and on the underlying value of  $K_\rho$ , which, of course, must be the same for different experiments in any specific material.

In order to see charge-spin separation, one must perform  $q$ - and  $\omega$ -resolved spectroscopy (or time-of-flight measurements). Photoemission spectroscopy is the first choice because it directly probes single-particle excitations [14]. With some approximations, it measures the imaginary part of the electronic Green's function, and Luttinger liquid theory predicts, cf. Fig. 2 [15,16]

$$\rho(q, \omega) = \frac{-1}{\pi} \text{Im} G(q + k_F, \omega + E_F) \sim (\omega - v_\sigma q)^{\alpha-1/2} |\omega - v_\sigma q|^{(\alpha-1)/2} (\omega + v_\rho q)^{\alpha/2}. \quad (6)$$

One finds *two dispersing singularities* (with interaction dependent exponents; for  $\alpha$ , cf. above) which demonstrates that the electron ejected from the material is composed out of two more elementary excitations. By  $q$ -integration, one can obtain the density of states,  $N(\omega) \sim |\omega|^\alpha$ , and by  $\omega < 0$ -integration  $n(k)$ . A practical comment: the easy part is the calculation of the Green's function in bosonization.



**FIGURE 2.** Spectral functions of a Luttinger liquid. The three signals represent the holon, the spinon, and the shadow bands (left to right). Left panel: weak/short-range interactions,  $\alpha = 1/8$  ( $K_\rho = 1/2$ ). Right panel: strong/long-range interactions,  $\alpha = 1.5$  ( $K_\rho = 1/8$ ).

The difficult part is the Fourier transformation if, e.g., the result must satisfy sum rules. Charge-spin separation is also visible, though with different exponents, in dynamical density and spin correlation at  $q \approx 2k_F$  [16]. They can be measured, in principle, by EELS, inelastic neutron scattering, or Raman scattering.

## STABILITY OF LUTTINGER LIQUIDS

Luttinger liquid theory crucially relies on one-dimensionality. Moreover, most of our discussion was for  $T = 0$ , and ignored phonons, lattice effects, impurities, etc. Are these factors detrimental to Luttinger liquids? In many cases, the answer will depend on the scales one considers.

Finite temperature is not a problem, and the correlation functions discussed above can be calculated for  $T > 0$ . Quite generally, however, divergences will be cut off by  $T$  whenever  $T > \omega, v_\nu|q|, \dots$ . Also charge-spin separation will be masked in the spectral function when  $(v_\rho - v_\sigma)q < T$  [17].

Interchain tunneling will introduce 3D effects. Depending on the on-chain interactions, it will either produce a crossover to a Fermi liquid (weak interactions), or to a long-range ordered 3D insulating or superconducting phase (strong interactions) [18]. In any event, *a Luttinger liquid is unstable towards 3D coupling at low enough temperature (scales)*. However, on high enough scales, it will be unaffected by 3D coupling, and coming from there, one will encounter a crossover temperature below which 3D correlations will build up, and 1D physics will be strongly modified. At still lower temperature, a phase transition may take place into a long-range ordered 3D state. When going to a Fermi liquid, the crossover is gradual, and Luttinger-like spectral functions can be observed somewhat off the Fermi energy [19].

Other sources of concern are phonons and lattice effects. Various studies of phonons coupled to Luttinger liquids have shown that depending on details of the electron-electron and electron-phonon interactions, a Luttinger liquid may remain stable, though renormalized, when phonons are added [8]. Alternatively, the electron-phonon interaction could lead to the opening of a spin gap, and thereby destabilize the Luttinger liquid. This situation is described by a different model due to Luther and Emery, but the correlation functions continue to carry certain remnants of Luttinger physics, like non-universal power laws stemming from the gapless charges (the system remains conducting), and charge-spin separation [20].

When the crystal lattice is important (commensurate band filling), the system may become insulating. For a 1D band insulator, Luttinger liquid physics is expected to be lost completely, although not much is known firmly [21]. More interesting is the case of a Mott insulator, brought about by electronic correlations. However even here, charge-spin separation still is seen, e.g. in photoemission both in theory [20] and in experiments on  $\text{SrCuO}_2$  [22]. Moreover, far above the (charge or spin) gaps, they should no longer influence the physics, and genuine Luttinger liquid behavior is expected there.

## HOW TO APPLY LUTTINGER LIQUID THEORY?

We discuss the example of the organic conductor TTF-TCNQ, starting from a recent photoemission study [23]. Photoemission shows a valence band signal whose dispersion is in qualitative agreement with a simple Hückel band structure, along the 1D chains, and no dispersion perpendicular. Two discrepancies between the data and a quasi-particle picture can be resolved in a Luttinger liquid picture: (i) the experimental dispersions are bigger than those expected from the density of states at the Fermi level within the Hückel band structure. (ii) The lineshapes are anomalous in that the signal on the TTF band has a tail reaching up to  $E_F$  at all  $k$ , while it has a low-energy shoulder with little dispersion on TCNQ. Both findings are consistent with Luttinger liquid spectral functions, with  $K_\rho \ll 1/2$  on TTF, and with  $1/2 < K_\rho < 1$  on TCNQ. Also recall that Luttinger liquids show more dispersion than Fermi liquids because of the upward renormalization of  $v_\rho$  by interactions.

Is this assignment consistent with other information? It is consistent for the TTF band. In fact, the magnetic susceptibility is rather independent of temperature [24], and diffuse X-ray scattering observes strong  $4k_F$  density fluctuations at high temperatures [10]. It is not consistent, however, for the TCNQ band where the susceptibility is strongly  $T$ -dependent, with an activated shape. This can be taken as an indication of a spin gap, and suggests that the Luther-Emery model might be a better choice. The observation of  $2k_F$  density fluctuations on TCNQ is consistent with both assignments, and the spectral function of the Luther-Emery model can at least qualitatively describe the data.

Evidence for or against such hypotheses must come from further experiments. Optics shows a far-IR pseudogap [25]. However, the consistency of the mid-IR conductivity with (5) should be checked. Also notice that [26]  $\rho(T) \sim T$ . One might look into the temperature dependence of the spin conductivity in view of theories discussing the manifestation of charge-spin separation in transport properties [27]. NMR could look for the  $T^{K_\rho}$ -term of Eq. (4), and Raman scattering could show if the values of  $v_\nu$  measured through two-particle excitations superpose to the dispersions of the photoemission peaks. If successful, the Luttinger liquid theory will provide a consistent phenomenology for the low-energy properties of this material, and will have predictive power for future experiments.

## ASPECTS OF MESOSCOPIC SYSTEMS

Due to the small sample size, boundary conditions become of importance, and may dominate the physics. As an example, for a quantum wire, the conductance is given by  $G_n = 2nK_\rho e^2/h$  where  $n$  is the number of conducting channels [28]. When the wire is coupled to Fermi liquid leads, however, the interaction renormalization is absent [29], and  $G_n = 2ne^2/h$  – a boundary effect!

The influence of isolated impurities on transport, or tunneling through quantum point contacts, is an important problem [28,30]. At higher temperatures (voltages), there will be corrections to the (differential) conductance  $\delta G \sim T^{K_\rho-1}$ , resp.  $\delta(dI/dV) \sim V^{K_\rho-1}$ . With repulsive interactions and at low energy scales, an impurity will cut the quantum wire into two segments with only a weak link between them. In this case, the conductance, resp. differential conductance, vary as  $G(T) \sim T^{K_\rho^{-1}-1}$ , resp.  $dI/dV \sim V^{K_\rho^{-1}-1}$ . The physical origin of this effect is the establishment of a strong Friedel oscillation around the impurity which will increasingly backscatter the electrons at lower energy scales.

An impurity can therefore be assimilated with open boundary conditions. This identifies the exponents just described as members of a larger class of *boundary critical exponents*. Quite generally, 1D interacting fermions with open boundaries and gapless excitations form a *bounded Luttinger liquid* state, rather similar to ordinary Luttinger liquids but with a different set of exponents and scaling relations [30]. The  $K_\nu$  are properties of the Hamiltonian, and therefore independent of boundary conditions. The correlation functions, and their exponents, however depend on boundary conditions.

A particularly nice experiment demonstrating this relation, has been performed on carbon nanotubes [31]. With different preparations, it is possible to tunnel electrons from electrodes either into the end of nanotubes, or into their bulk. In the first case, conductance and differential conductance measure the power-laws just described for tunneling through a weak link, while for tunneling into the bulk, they measure the bulk density of states, described in the context of photoemission. The exponents differ slightly from those given here because of the peculiar band structure of the tubes and because the electrons tunnel from a Luttinger liquid into a normal metal [9]. The remarkable result of this work is that the various experiments can be described in terms of a *single coupling constant*  $K_\rho \sim 0.28$ .

## OPEN QUESTIONS

The preceding discussion may suggest that one-dimensional fermions are completely understood, at least theoretically. However, many important questions remain open, both in theory and experiment. I now list a few of them.

One important problem relates to scales. While common folklore states that Luttinger liquids form on energy scales between the electronic bandwidth or the typical interaction energy, whichever is smaller, on the high-energy side, and the 3D crossover temperature on the low-energy side, it is not known with certainty if all predicted properties can indeed be observed in that range. Can both power laws and charge-spin separation be observed over the entire range? Some studies seem to suggest that, in the 1D Hubbard model, the Green's function power laws may be restricted to smaller scales [32]. Are these ranges the same for all correlations, or do they depend on the specific function considered? Do they depend on the specific Hamiltonian considered, e.g. on the interaction strength and range, and how?



Concerning mesoscopic systems, only Luttinger liquids with open boundaries are thoroughly characterized. It is conceivable that other boundary conditions (Fermi liquid leads, boundary fields or spins, superconductors) lead to new sets of critical exponents.

What is the spectral weight associated with Luttinger liquid physics in any given microscopic model, or in any given experimental system? Can one measure, in analogy to the quasi-particle residue  $Z$  in Fermi liquids, the weight of the coherent spin and charge modes, with respect to the incoherent contributions to the Green's function, or to any other correlation function? How sure can we be that this weight is sufficiently high, so that experiments (e.g. photoemission) actually see these excitations, and not just incoherent contributions or bare high-energy excitations? Is the high-energy physics, far from the Fermi surface, necessarily non-universal and strongly material- (model-) dependent, as is often claimed?

In the same way, the interpretation of some experiments, e.g. photoemission, rests crucially on the appropriateness of simple Hückel-type bandstructures. However, the materials investigated to date, are very complex, and there is no guarantee that these methods are appropriate. There are two ways out. (i) More sophisticated band structure methods become more performing as the computer power increases, and should attack the complex materials of interest here [33]. (ii) One might also look at novel structures where extremely simple 1D materials can be produced. One example for this direction are gold wires deposited on a vicinal Si(111)5x1 surface, where photoemission may have detected evidence for charge-spin separation and Luttinger liquid behavior [34]. (iii) In mesoscopic wires, both on semiconductor and tube base, we would love to have spectroscopic experiments made feasible which probe the dynamics of the elementary excitations beyond transport. As a first step, the study of “noise” might provide interesting insights [35].

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